Schurman RO1C Model - Discounts for Reliance on One Customer

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ABC Company produces and sells widgets. Variable cost (raw materials, direct labor, etc.) per widget is \$4.00. Each widget is sold for \$10.00 so contribution margin is 60% (profit before fixed costs divided by selling price). If a major customer is lost then revenue, variable costs, fixed costs and capital expenditures may all be affected. If ABC Company's variable cost is truly variable then \$4.00 in variable cost is saved for every \$10.00 in revenue lost such that contribution margin is unaffected. Fixed costs may be partially affected or not affected at all. Investment in assets such as accounts receivable, inventory and fixed assets (capital expenditures) should decrease.

From a valuation perspective, how should the potential for loss of a major customer affect the valuation? ABC's customers come and go on a regular basis. If each of ABC's customers represent a relatively small percentage of revenue then we could assume that the projected revenue growth rate includes the loss of existing customers plus the addition of new ones. If a significant portion of ABC's revenue is attributable to one major customer and the probability of replacing that customer is small then we should provide for the possibility of losing this customer in our valuation. To not provide for that possibility would overstate company value. In the credit markets it is often said that "every customer defaults, it's only a matter of when" (for a AAA bond the probability of default is approximately 2 basis points, or once every 5,000 years). We may say the same for ABC's customer base. ABC's major customer will be lost, it's only a matter of when.

The buyer of ABC will definitely want to incorporate the potential for loss of this major customer into his valuation. The customer could be lost for a variety of reasons including switch to another supplier, bankruptcy, change in control (ABC's customer or ABC itself) and exiting a line of business. ABC's current owners may feel very confident that the customer will not leave but the potential for loss must also also be viewed from the perspective of the buyer.

Legend of Symbols

A_t	=	Total assets at time t
C	=	Revenue contribution margin
F_t	=	Annualized fixed costs at time t
M_R	=	Percentage of total revenue derived from major customer
M_F	=	Percentage of total fixed costs attributable to major customer
R_t	=	Revenue at time t
S_t	=	Probability that customer suvives (i.e. is not lost) by time t
V_R	=	Present value of revenue contribution stream
V_F	=	Present value of fixed costs stream
V_X	=	Present value of capital expenditures stream
p	=	Probability of loss of major customer in any time period
t	=	Time period number
μ	=	Periodic revenue growth rate
ι	=	Periodic inflation rate
κ	=	Periodic discount rate
λ	=	Target ratio of assets to revenue

Probability of Loss

We will define survival as whether or not the customer is lost by time t. If the customer is not lost then the customer survives. If the customer is lost then the customer does not survive. If the probability that the customer is lost during any time period t is p then the probability of survival at time t is...

$$S_t = (1-p)^t \tag{1}$$

Note: Event modeling is a subject unto itself. For more information on this topic see paper titled 'Using A Transition Matrix To Model Events'.

Present Value of Revenue Contribution

For our valuation we will assume that revenue grows at the constant rate μ. The equation for the present value of revenue contribution at $t = 0$ is...

$$V_R = \sum_{t=1}^{T} CR_0 (1+\mu)^t (1+\kappa)^{-t} \tag{2}$$

The present value equation (2) above assigns a zero probability to the loss of a major customer. The random variate a_t will represent a multiple of revenue. If the customer is not lost by time t then the value of the random variate is one. If the customer is lost by time t then the value of the random variate is the percentage that revenue would fall should this customer be lost. The probability that $a_t = 1$ is $(1-p)^t$. The probability that $a_t = 1 - M_R$ is $1-(1-p)^t$. The present value equation modeled as a stochastic process is now...

$$V_R = CR_0 \sum_{t=1}^{T} (1+\mu)^t (1+\kappa)^{-t} a_t \tag{3}$$

The expected value of the random variate a_t is...

$$\begin{aligned} \mathbb{E}\Big[a_t\Big] &= \Big[1 \times (1-p)^t\Big] + \Big[(1-M_R) \times (1-(1-p)^t)\Big] \\ &= (1-M_R) + M_R(1-p)^t \end{aligned} \tag{4}$$

Present value equation (3) modeled as an expectation is...

$$\begin{aligned} \mathbb{E}\Big[V_R\Big] &= CR_0 \sum_{t=1}^{T} (1+\mu)^t (1+\kappa)^{-t}\, \mathbb{E}\Big[a_t\Big] \\ &= CR_0 \sum_{t=1}^{T} (1+\mu)^t (1+\kappa)^{-t} \Big[(1-M_R) + M_R(1-p)^t\Big] \\ &= CR_0 \Big[(1-M_R) \sum_{t=1}^{T} (1+\mu)^t (1+\kappa)^{-t} + M_R \sum_{t=1}^{T} (1+\mu)^t (1-p)^t (1+\kappa)^{-t}\Big] \end{aligned} \tag{5}$$

We will make the following simplifying definitions...

$$\theta_R = \frac{(1+\mu)}{(1+\kappa)} \text{ ...and... } \phi_R = \frac{(1+\mu)(1-p)}{(1+\kappa)} \tag{6}$$

After substituting the above simplifying definitions into equation (5) we can rewrite that equation as...

$$\mathbb{E}\Big[V_R\Big] = CR_0 \Big[(1-M_R) \sum_{t=1}^{T} \theta_R^t + M_R \sum_{t=1}^{T} \phi_R^t\Big] \tag{7}$$

If the revenue growth rate is less than the discount rate then the two summations in equation (7) are geometric series. To take advantage of this property we must start the summations at $t = 0$ rather than $t = 1$. After making this adjustment the equation becomes...

$$\mathbb{E}\Big[V_R\Big] = CR_0 \Big[(1-M_R) \Big\{ \sum_{t=0}^{T} \theta_R^t - 1 \Big\} + M_R \Big\{ \sum_{t=0}^{T} \phi_R^t - 1 \Big\}\Big] \tag{8}$$

Noting that equation (8) now contains two geometric series we can replace the two summations. After making these replacements the equation becomes...

$$\mathbb{E}\bigg[V_R\bigg] = CR_0\bigg[(1-M_R)\bigg\{\frac{1-\theta_R^{T+1}}{1-\theta_R}-1\bigg\}+M_R\bigg\{\frac{1-\phi_R^{T+1}}{1-\phi_R}-1\bigg\}\bigg] \tag{9}$$

Since we will be modeling present value as a perpetuity then T will equal infinity. Since θ and ϕ are both less than one then these two variables raised to the power of infinity become zero. After replacing θ_R^{T+1} and ϕ_R^{T+1} with zero equation (9) becomes...

$$\mathbb{E}\bigg[V_R\bigg] = CR_0\bigg[(1-M_R)\bigg\{\frac{\theta_R}{1-\theta_R}\bigg\}+M_R\bigg\{\frac{\phi_R}{1-\phi_R}\bigg\}\bigg] \tag{10}$$

Present Value of Fixed Costs

For our valuation we will assume that fixed costs grow at the constant rate ι. The equation for the present value of fixed costs at $t=0$ is...

$$V_F = \sum_{t=1}^{T} F_0(1+\iota)^t(1+\kappa)^{-t} \tag{11}$$

The present value equation (11) above assigns a zero probability to the loss of a major customer. The random variate b_t will represent a multiple of fixed costs. If the customer is not lost by time t then the value of the random variate is one. If the customer is lost by time t then the value of the random variate is the percentage that fixed costs would fall should this customer be lost. The probability that $b_t = 1$ is $(1-p)^t$. The probability that $b_t = 1 - M_F$ is $1-(1-p)^t$. The present value equation modeled as a stochastic process is now...

$$V_F = F_0\sum_{t=1}^{T}(1+\iota)^t(1+\kappa)^{-t}b_t \tag{12}$$

The expected value of the random variate b_t is...

$$\begin{aligned}\mathbb{E}\bigg[b_t\bigg] &= \bigg[1\times(1-p)^t\bigg]+\bigg[(1-M_F)\times(1-(1-p)^t)\bigg]\\ &= (1-M_F)+M_F(1-p)^t\end{aligned} \tag{13}$$

Present value equation (12) modeled as an expectation is...

$$\begin{aligned}\mathbb{E}\bigg[V_F\bigg] &= F_0\sum_{t=1}^{T}(1+\iota)^t(1+\kappa)^{-t}\,\mathbb{E}\bigg[b_t\bigg]\\ &= F_0\sum_{t=1}^{T}(1+\iota)^t(1+\kappa)^{-t}\,\bigg[(1-M_F)+M_F(1-p)^t\bigg]\\ &= F_0\bigg[(1-M_F)\sum_{t=1}^{T}(1+\iota)^t(1+\kappa)^{-t}+M_F\sum_{t=1}^{T}(1+\iota)^t(1-p)^t(1+\kappa)^{-t}\bigg]\end{aligned} \tag{14}$$

We will make the following simplifying definitions...

$$\theta_F = \frac{(1+\iota)}{(1+\kappa)}\ \text{...and...}\ \phi_F = \frac{(1+\iota)(1-p)}{(1+\kappa)} \tag{15}$$

If the inflation rate is less than the discount rate then we can use the mathematical procedures outlined in equations (7) to (9) above to write the expected present value of fixed costs equation. This equation is...

$$\mathbb{E}\bigg[V_F\bigg] = F_0\bigg[(1-M_F)\bigg\{\frac{\theta_F}{1-\theta_F}\bigg\}+M_F\bigg\{\frac{\phi_F}{1-\phi_F}\bigg\}\bigg] \tag{16}$$

Present Value of Capital Expenditures

If we have projected positive revenue growth then we must provide for capital expenditures. The equation for total assets at any time t as a function of revenue and modeled as a stochastic process via equation (3) above is...

$$A_t = \lambda R_0 (1+\mu)^t a_t \tag{17}$$

The present value of capital expenditures is the sum of the discounted value of the expected changes in assets over time. The equation for the present value of capital expenditures at time $t = 0$ is...

$$V_X = \sum_{t=1}^{T} \mathbb{E}\Big[\Delta A_t\Big](1+\kappa)^{-t} \tag{18}$$

The expected change in assets at any time t is assets at time t minus assets at time $t-1$. The equation for the expected change in assets at any time t is...

$$\begin{aligned}
\mathbb{E}\Big[\Delta A_t\Big] &= \lambda R_0 (1+\mu)^t \,\mathbb{E}\Big[a_t\Big] - \lambda R_0 (1+\mu)^{t-1}\,\mathbb{E}\Big[a_{t-1}\Big] \\
&= \lambda R_0 (1+\mu)^{t-1}\Big[\mathbb{E}\Big[a_t\Big](1+\mu) - \mathbb{E}\Big[a_{t-1}\Big]\Big] \\
&= \lambda R_0 (1+\mu)^{t-1}\Big[\Big[(1-M_R) + M_R(1-p)^t\Big](1+\mu) - \Big[(1-M_R) + M_R(1-p)^{t-1}\Big]\Big] \\
&= \lambda R_0 (1+\mu)^{t-1}\Big[\mu(1-M_R) + M_R(1-p)^{t-1}((1+\mu)(1-p)-1)\Big]
\end{aligned} \tag{19}$$

After replacing the expected change in assets in equation (18) with equation (19) the equation for the present value of capital expenditures becomes...

$$\begin{aligned}
\mathbb{E}\Big[V_X\Big] &= \sum_{t=1}^{T} \lambda R_0 (1+\mu)^{t-1}\Big[\mu(1-M_R) + M_R(1-p)^{t-1}((1+\mu)(1-p)-1)\Big](1+\kappa)^{-t} \\
&= \lambda R_0 (1+\mu)^{-1}\Big[\sum_{t=1}^{T} \mu(1-M_R)\theta_R + \sum_{t=1}^{T} M_R(1-p)^{t-1}((1+\mu)(1-p)-1)\theta_R\Big] \\
&= \lambda R_0 (1+\mu)^{-1}\Big[\mu(1-M_R)\sum_{t=1}^{T}\theta_R + M_R((1+\mu)(1-p)-1)\sum_{t=1}^{T}(1-p)^{t-1}\theta_R\Big]
\end{aligned} \tag{20}$$

We can break equation (20) into two parts, equation V_{X_a} and equation V_{X_b}, and evaluate them separately. This equation broken into parts is...

$$\mathbb{E}\Big[V_{X_a}\Big] = \lambda R_0 (1+\mu)^{-1}\Big[\mu(1-M_R)\sum_{t=1}^{T}\theta_R\Big] \tag{21}$$

$$\mathbb{E}\Big[V_{X_b}\Big] = \lambda R_0 (1+\mu)^{-1}\Big[M_R((1+\mu)(1-p)-1)\sum_{t=1}^{T}(1-p)^{t-1}\theta_R\Big] \tag{22}$$

The solution to equation V_{X_a} is...

$$\begin{aligned}
\mathbb{E}\Big[V_{X_a}\Big] &= \lambda\mu R_0 (1-M_R)(1+\mu)^{-1}\sum_{t=1}^{T}\theta_R \\
&= \lambda\mu R_0 (1-M_R)(1+\mu)^{-1}\Big\{\sum_{t=0}^{T}\theta_R - 1\Big\} \\
&= \lambda\mu R_0 (1-M_R)(1+\mu)^{-1}\Big\{\frac{1-\theta_R^{T+1}}{1-\theta_R} - 1\Big\}
\end{aligned} \tag{23}$$

As T goes to infinity equation V_{X_a} becomes...

$$\mathbb{E}\Big[V_{X_a}\Big] = \lambda\mu R_0(1-M_R)(1+\mu)^{-1}\Bigg\{\frac{\theta_R}{1-\theta_R}\Bigg\} \tag{24}$$

The solution to equation V_{X_b} is...

$$\begin{aligned}
\mathbb{E}\Big[V_{X_b}\Big] &= \lambda R_0 M_R(1+\mu)^{-1}((1+\mu)(1-p)-1)\sum_{t=1}^{T}(1-p)^{t-1}\theta_R \\
&= \lambda R_0 M_R(1+\mu)^{-1}((1+\mu)(1-p)-1)\sum_{t=1}^{T}(1-p)^{-1}(1-p)^{t}\theta_R \\
&= \lambda R_0 M_R(1+\mu)^{-1}(1-p)^{-1}((1+\mu)(1-p)-1)\sum_{t=1}^{T}\phi_R \\
&= \lambda R_0 M_R(1-(1+\mu)^{-1}(1-p)^{-1})\Bigg\{\sum_{t=0}^{T}\phi_R - 1\Bigg\} \\
&= \lambda R_0 M_R(1-(1+\mu)^{-1}(1-p)^{-1})\Bigg\{\frac{1-\phi_R^{T+1}}{1-\phi_R} - 1\Bigg\} \qquad (25)
\end{aligned}$$

As T goes to infinity equation V_{X_b} becomes...

$$\mathbb{E}\Big[V_{X_b}\Big] = \lambda R_0 M_R(1-(1+\mu)^{-1}(1-p)^{-1})\Bigg\{\frac{\phi_R}{1-\phi_R}\Bigg\} \tag{26}$$

Summary

The present value of after-tax revenue contribution via equation (10) is...

$$\mathbb{E}\Big[V_R\Big] = CR_0\Bigg[(1-M_R)\Bigg\{\frac{\theta_R}{1-\theta_R}\Bigg\} + M_R\Bigg\{\frac{\phi_R}{1-\phi_R}\Bigg\}\Bigg] \times (1-taxrate) \tag{27}$$

The present value of after-tax fixed costs via equation (16) is...

$$\mathbb{E}\Big[V_F\Big] = F_0\Bigg[(1-M_F)\Bigg\{\frac{\theta_F}{1-\theta_F}\Bigg\} + M_F\Bigg\{\frac{\phi_F}{1-\phi_F}\Bigg\}\Bigg] \times (1-taxrate) \tag{28}$$

The present value of capital expenditures via equations (24) and (26) is...

$$\mathbb{E}\Big[V_X\Big] = \lambda R_0\Bigg[\mu(1-M_R)(1+\mu)^{-1}\Bigg\{\frac{\theta_R}{1-\theta_R}\Bigg\} + M_R(1-(1+\mu)^{-1}(1-p)^{-1})\Bigg\{\frac{\phi_R}{1-\phi_R}\Bigg\}\Bigg] \tag{29}$$

Company value = Equation (27) minus Equation (28) minus Equation (29)

Note: Since the loss of a major customer is a future event, the higher the discount rate the less of an effect the loss of the major customer will have on company value.

A Hypothetical Case

ABC company has a reliance on one major customer. This customer accounts for 60% of revenue and it is estimated that if this customer were lost fixed costs could be cut by 25%. The business appraiser wants to calculate a discount due to reliance on one customer.

Assumptions:

Current revenue	=	1000
Current fixed costs	=	300
Contribution margin	=	70%
Revenue growth	=	4%
Inflation rate	=	3%
Discount rate	=	18%
Probability of loss	=	10%
Ratio assets to revenue	=	40%
Income tax rate	=	20%

Preliminary Calculations:

$$
\begin{aligned}
\theta_R &= (1+\mu)^1(1+\kappa)^{-1} &&= (1+0.04)^1(1+.18)^{-1} &&= 0.8814 \\
\phi_R &= (1+\mu)^1(1-p)^1(1+\kappa)^{-1} &&= (1+0.04)^1(1-.10)^1(1+.18)^{-1} &&= 0.7932 \\
\theta_F &= (1+\iota)^1(1+\kappa)^{-1} &&= (1+0.03)^1(1+.18)^{-1} &&= 0.8729 \\
\phi_F &= (1+\iota)^1(1-p)^1(1+\kappa)^{-1} &&= (1+0.03)^1(1-.10)^1(1+.18)^{-1} &&= 0.7856
\end{aligned}
$$

Components of Value:

The present value of after-tax revenue contribution via equation (27) is...

$$
\begin{aligned}
\mathbb{E}\Big[V_R\Big] &= (.70)(1000)\Bigg[(1-0.60)\left\{\frac{0.8814}{1-0.8814}\right\}+(0.60)\left\{\frac{0.7932}{1-0.7932}\right\}\Bigg]\times(1-0.20) \\
&= 2953
\end{aligned}
$$

The present value of after-tax fixed costs via equation (28) is...

$$
\begin{aligned}
\mathbb{E}\Big[V_F\Big] &= (300)\Bigg[(1-0.25)\left\{\frac{0.8729}{1-0.8729}\right\}+(0.250\left\{\frac{0.7856}{1-0.7856}\right\}\Bigg]\times(1-0.20) \\
&= 1456
\end{aligned}
$$

The present value of capital expenditures via equation (29) is...

$$
\begin{aligned}
\mathbb{E}\Big[V_X\Big] &= (.40)(1000)\Bigg[(0.04)(1-0.60)(1+0.04)^{-1}\left\{\frac{0.8814}{1-0.8814}\right\}+(0.60)(1-(1+0.04)^{-1}(1-0.10)^{-1})\left\{\frac{0.7932}{1-0.7932}\right\}\Bigg] \\
&= -17
\end{aligned}
$$

Company Value and Discount:

Company value with probability of loss equal to 10% = 2953 - 1456 -(-17) = 1514

Company value with probability of loss equal to zero = 2398 *

Discount = (2398 - 1514) / 2398 = 37%

* This approximates the standard cap rate value when income is divided by discount rate minus growth rate.